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Six Dimensional Topological Gravity and the Cosmological Constant Problem

G. Bonelli and A.M. Boyarsky

Spinoza Institute, University of Utrecht
Leuvenlaan 4, 3584 CE Utrecht, The Netherlands
bonelli@phys.uu.nl boyarsky@phys.uu.nl

Abstract: We formulate a topological theory in six dimensions with gauge group $SO(3,3)$ which reduces to gravity on a four dimensional defect if suitable boundary conditions are chosen. In such a framework we implement the reflection automorphism of $SO(3,3)$ as a $\mathbf{Z}_2$ symmetry which forbids the appearance of a gravitational cosmological constant. Some temptative speculations are presented also for the possible inclusion of the matter contribution at a full quantum level.

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1 Introduction

The fine tuning of the cosmological constant is one of the open problems in modern theoretical physics [1]. In few words, the Hilbert-Einstein action $S_{HE} = \frac{1}{16\pi G} \int R\sqrt{g} d^4x$ admits a natural extension to $S_{HE} + \Lambda \int \sqrt{g} d^4x$ for any constant real Λ. From the gravitational point of view the natural scale for Λ is the gravitational inverse volume factor $\frac{1}{G^2} \sim M^4_{Plank}$.

On another side, a sensible non zero additive contribution to the cosmological constant is expected on general grounds from the quantum theory of the Standard model and to be of the order M^4_{SM} as a vacuum energy.

Actually none of these two scales are observed. The present experimental data [2] in fact predict Λ to be absolutely negligible with respect to both the natural gravitational inverse volume and the Standard model one.

The fine tuning of the cosmological constant represents then a problem since it seems unnatural that two contributions depending on two independent scales do cancel a priori.

A usual mechanism in field theory to tune to zero a parameter is to impose a symmetry which prohibits it. A promising candidate for this role is supersymmetry which naturally demands zero vacuum energy. One obvious difficulty with supersymmetry from this prospective is that it should be broken at the Standard Model scale while the cosmological constant is still negligibly small.

On another side, in recent times, a phenomenological new framework arose in considering the possible pros in having extra dimensions [3] where gravitational degrees of freedom can propagate. In this letter we will begin our analysis by considering a different type of extra dimensions.

Let us describe our starting point by recalling a well known fact. Any theoretical physicist lives with its own signature for the space-time metric: depending on taste one can choose either $(-,+,+,+)$ or $(+,-,-,-)$. Both possibilities are equivalent with respect to S_{HE} up to a global sign reversal, but they differ if the cosmological term is added. The latter does not take any sign reversal and therefore one relates the two conventions by

$$S_{HE} + \Lambda \int \sqrt{g} \quad \rightarrow \quad -\left(S_{HE} - \Lambda \int \sqrt{g}\right) .$$

In this letter we will promote this convention choice to a $\mathbf{Z}_2$ symmetry which in principle could prohibit the cosmological term.

This could be achieved by embedding the relative local $SO(1,3)$ Lorentz group into a larger $SO(3,3)$ gauge group governing a six dimensional gravitational field theory. In such

an enlarged theory one can consider inequivalent embeddings of $SO(1,3)$ in $SO(3,3)$. These embeddings are in two classes which are exchanged by the signature changing operator S [1].

A first difficulty that one encounters in trying to perform such a construction is that, if the six dimensional field theory admits propagating degrees of freedom, from the point of view of an embedded four dimensional world the appearance of tachionic transverse excitations takes place. Therefore, to avoid this unwelcome circumstance, the most obvious possibility is to restrict the class of possible six dimensional theories to that of topological field theories in such a way that any propagation of degrees of freedom in its bulk space-time does not take place at all [4]. Let us here note that the appearance of a possible role played by topological field theories in a framework which could cancel the gravitational cosmological constant was already advocated in [5]. In this letter we will build a minimal model which fulfills all the above properties.

After it became clear that three dimensional gravity can be reformulated as a Chern-Simons theory [6] there has been a lot of attempts to understand if four dimensional gravity has something to do with some other topological theory. Here we present a construction of classical four dimensional gravity as a six dimensional topological theory which induces dynamical degrees of freedom on a four dimensional defect if properly coupled to it.

Then, using this result, we will describe a mathematical realization of a discrete $\mathbf{Z}_2$ symmetry which prohibits the appearance of a non zero cosmological constant in classical gravity theory. We will finally implement such a symmetry quantum mechanically.

Let us take henceforth a minimal perspective in which we try to realize such an higher theory as a topological gauge theory in six dimensions with gauge group $SO(3,3)$ and just a $so(3,3)$ valued connection $\mathcal{A}$ as a fundamental field. In other words, we consider a topological gravity theory in six dimensions consisting only of a spin connection and not of a couple spin connection and sechs-bein.

A second difficulty that one meets in this model is related to the implementation of the $\mathbf{Z}_2$ symmetry. The group $SO(3,3)$ admits a $\mathbf{Z}_2$ automorphism S which exchanges $\eta \to \eta^{\mathrm{S}} = -\eta$ and acts as an outer automorphism on the group. Its invariant subgroup is isomorphic to $GL(3)$. Therefore, if one tries to restrict naively any $so(3,3)$ six dimensional connection in an S-invariant way, one can at most obtain a four dimensional $GL(3)$ gauge group which, unfortunately, can not contain any $SO(1,3)$ gauge subgroup. This fact implies that the

[1] As we will see, a very important point is represented by the fact that S is an outer automorphism of $SO(3,3)$ in the sense that S itself is not a $SO(3,3)$ group element. This is explained in more details in the Appendix.

simplest embedding procedure has to implement the signature changing operator S on a couple of S-conjugate configurations $\mathcal{A}^{(\sigma)}$, with $\sigma = \pm 1$, whose content will be properly fixed afterwards to obtain a symmetric configuration [2].

The main idea that we will work out here is the following. We consider a topological field theory in a six dimensional space Y with boundary such that no dynamics is induced on the local degrees of freedom in the interior of Y. Then non trivial dynamics can only take place at the boundary. We couple the boundary to the bulk fields by specifying their boundary conditions. What remains is then a dynamical theory on the boundary for the unfixed part of restricted bulk fields. We can generate such a situation in a suitable regularized version of $Y \setminus W$, where W is a four dimensional submanifold of Y. In this case the boundary theory can be then resolved to a four dimensional theory on W.

In the following we will first give a mathematical account for the situation described above and then, in a subsequent section, we will explain how to use this mathematical construction for classical gravity and how to include in the model any quantum matter contribution.

2 From six to four dimensions

Let Y be a (compact) six dimensional manifold and W a four dimensional sub-manifold embedded into Y. Let τ be the Poincare' dual of W, i.e. a closed 2-form on Y such that $\int_Y \tau \wedge O^{(4)} = \int_W O^{(4)}|_W$ for all 4-forms [3] in Y. Really τ is defined up to a full differential and so, being a cohomology class, we can choose various representatives for it. Following [7] we can choose $\tau = d\left(\rho \frac{d\theta}{2\pi}\right)$, where ρ is a bump function, which is equal to zero on W and to -1 far enough from it while θ is a local angular coordinate along the transverse S^1. Let $\mathcal{A}_{AB} = A^{\mu}_{AB} dx_{\mu}$ be a connection one form for a given $SO(3,3)$ bundle $\mathcal{X}$ on Y and $\mathcal{F}_{AB} = d\mathcal{A}_{AB} + \mathcal{A}_{AC} \wedge \mathcal{A}^{C}_{B}$ be its curvature two form valued in $so(3,3)$.

If Y is compact, the functional $I_6 = c \int_Y \epsilon^{ABCDEF} \mathcal{F}_{AB} \wedge \mathcal{F}_{CD} \wedge \mathcal{F}_{EF} = c \int_Y \mathcal{F}^3$ is independent on the particular connection and defines an invariant of the bundle. Under a properly chosen c prefactor, I_6 is, in this case, integer valued.

[2] Let us note that an apparently different way could be tried by embedding the problem in a complexified gauge group as $SO(3,3) \to SO(6,\mathbf{C})$ where the signature changing operator would act as an inner automorphism singling out a fixed complex structure in $SO(6,\mathbf{C})$. This again naturally leads to a similar doubling of the degrees of freedom.

[3] Locally it just means that if, e.g., W is embedded like $x_1 = x_2 = 0$ (for some coordinates $x_1 \dots x_6$), we can choose $\tau = dx_1 dx_2 \delta(x_1)\delta(x_2)$.

If Y has instead a boundary, this is no more true. In this case I_6 depends upon the value of the connection on the boundary of Y. To see this, for example, notice that if the bundle is trivial we can rewrite $I_6 = \int_{\partial Y}[C - S]$ as the integral of the corresponding Chern-Simons form on the boundary.

To adopt the above construction to the case of our interest, we have to make some more preliminary steps. Fix an auxiliary euclidean metric γ in Y and consider the space of points in Y which have γ-distance less than a given $\varepsilon > 0$ from W. Call this space B_ε^W and $Y_\varepsilon = Y \setminus B_\varepsilon^W$ its complement in Y. Y_ε is an ε-regularization for $Y \setminus W$. Notice that the disk ∂Y_ε is a realization of the total space of the normal bundle of W in Y [7]. This means that locally ∂Y_ε is like $W \times S^1$. From now on the limit $\varepsilon \to 0$ will be always implied in our formulas.

Now, to specify a coupling of the four dimensional defect to the bulk theory in this smoothed picture, we can choose some boundary conditions for $\mathcal{A}_{AB}$ at ∂Y_ε. For this, let's make the following index splitting along an $SO(2) \times SO(1,3)$ subgroup with $x, y, \ldots = 1, 2$ $SO(2)$ indices and $a, b, \ldots = 0, 1, 2, 3$ $SO(1,3)$ indices. Explicitly we define $\mathcal{A}_1^2 = \omega$, $\mathcal{A}_{a1} = e_a$, $\mathcal{A}_{a2} = \pi_a$ and $\mathcal{A}_{ab} = \omega_{ab}$. Let us remember that all these objects are 1-forms and so $\omega_{ab} = \omega_{ab}^\mu dx_\mu$ and so on [4].

Under these definitions, we get for the gauge curvature elements the following expressions

$$\mathcal{F}_{12} = \sigma d\omega - e^a \wedge \pi_a$$

$$\mathcal{F}_{a1} = De_a + \omega \wedge \pi_a$$

$$\mathcal{F}_{a2} = D\pi_a - \omega \wedge e_a$$

$$\mathcal{F}_{ab} = R_{ab} - \sigma(e_a \wedge e_b + \pi_a \wedge \pi_b)$$

where $\sigma = \pm 1$ is fixed by $\mathcal{A}_{12} = \sigma\omega$ and corresponds to the signature of the $SO(2)$ within $SO(3,3)$, $De_a = de_a + e_b \wedge \omega_a^b$, $D\pi_a = d\pi_a + \pi_b \wedge \omega_a^b$ and $R_{ab} = d\omega_{ab} + \omega_a^c \wedge \omega_{cb}$.

We specify our boundary conditions for the connection as the identification of the $SO(2)$ sub-bundle with the normal bundle of the defect $\mathcal{N}(W)$, which is $\mathcal{U}|_W = \sigma\mathcal{N}(W)$, as $d\omega = \tau$

[4] Let us also notice here that this splitting make sense only locally if the bundle $\mathcal{X}$ at W is irreducible. If it is instead reducible as $\mathcal{X}|_W = \mathcal{U} \oplus \mathcal{H}$, with $\mathcal{U}$ an $SO(2)$ bundle and $\mathcal{H}$ an $SO(1,3)$ bundle to be identified with the tangent bundle $T(W)$, then ω_{ab} is a connection for $\mathcal{H}$ and a singlet for $\mathcal{U}$, (e_a, π_a) are two frames for $\mathcal{H}$ and a doublet for $\mathcal{U}$ and ω is a singlet for $\mathcal{H}$ and a connection for $\mathcal{U}$.

[5] and fixing also one half of the transverse mixed connection components as $\pi_a|_W = 0$. After this boundary conditions are imposed, the degrees of freedom left on the defect are (e_a, ω_{ab}) which are valued in an $iso(1,3)$ subalgebra of our initial $so(3,3)$.

Now, we can calculate the actual value of the I_6 functional. Under the above decomposition we have

$$\mathcal{F}^3 = 6\left(\mathcal{F}_{12}\epsilon^{abcd}\mathcal{F}_{ab}\mathcal{F}_{cd} - 4\epsilon^{abcd}\mathcal{F}_{ab}\mathcal{F}_{c1}\mathcal{F}_{d2}\right) .$$

From this we recognize immediately that $\mathcal{F}^3$ is at most linear in ω and in $d\omega$ with generically a further inhomogeneous term. Let us recall that $\mathcal{F}^3$ is always [6] the total differential of the relative Chern-Simons form and in our case $\mathcal{F}^3 = d(\omega \wedge X^{(4)} + Z^{(5)})$, where $X^{(4)}$ and $Z^{(5)}$ are respectively a 4-form and a 5-form which are independent on ω. If the bundle is reducible in the sense above, the actual form of I_6 can be exactly calculated by Stoke's theorem as

$$I_6|_{b.c.} = c\int_{\partial Y_\varepsilon} \omega \wedge X^{(4)} + Z^{(5)} ,$$

where, with our boundary conditions,

$$X^{(4)} = \sigma\epsilon^{abcd}\left[R_{ab} - \sigma(e_a e_b)\right]\left[R_{cd} - \sigma(e_c e_d)\right]$$

and $Z^{(5)}$ vanishes. Choosing a representative for the W cohomology element and performing the $\varepsilon \to 0$ limit we can easily reduce this integral to a four dimensional one. Solve, in fact, $d\omega = \tau$ as $\omega = \rho\frac{d\theta}{2\pi} + d\phi$, for some scalar function ϕ, and perform the integral along the transverse circle [7]. In formulas we get that

$$I_6|_{b.c.} = -c\int_W X^{(4)} = -c\left[\sigma\int_W \epsilon^{abcd}R_{ab}\wedge R_{cd}\right.$$
$$\left. -2\int_W \epsilon^{abcd}R_{ab}\wedge e_c \wedge e_d + \sigma\int_W \epsilon^{abcd}e_a\wedge e_b \wedge e_c \wedge e_d\right] .$$

The result is then the H-E action on W augmented by a cosmological term and a term proportional to the Euler characteristic of W. Notice that the H-E action does not depend on the signature σ of the embedded $SO(2)$, while the other two terms do.

[5] Such that the freedom of choosing a particular ω-connection representative becomes the choice of a potential for τ, while the $SO(2)$ gauge invariance is the usual invariance of τ under a shift of an exact differential of the potential

[6] If the bundle is non trivial this is of course just a local statement, in the sense that the Chern-Simons form is not a well defined covariant form. In this section we will assume the bundle to be trivial, postponing the discussion about the non trivial bundle case to next section.

[7] To get $\int_{\partial Y_\varepsilon} d\phi \wedge X^{(4)} = 0$ one has really to use also the continuity properties at W of the involved fields.

3 The symmetric setting

The construction that we considered in the last section is not symmetric under the operation of changing the signature of the embedding of the normal bundle. One possible way to have a symmetric situation is to double the gauge bundle. This way we are able now to couple our defect symmetrically with respect to S-conjugation.

Let us therefore introduce two $SO(3,3)$ bundles $\mathcal{X}_\sigma$, labeled by $\sigma = \pm 1$, on Y and their corresponding connection one forms $\mathcal{A}^{(\sigma)}_{AB}$. Then we consider as a total action the sum of the two $\int_{Y_\varepsilon} \left(\mathcal{F}^{(\sigma)}\right)^3$ terms under the following boundary conditions. We assume the two bundles to be reducible to $\mathcal{U}_\sigma \oplus \mathcal{H}$ where $\mathcal{H}$ is a common $SO(1,3)$ factor and $\mathcal{U}_\sigma$ are two $SO(2)$ bundles such that $\mathcal{U}_\sigma|_W = \sigma \mathcal{N}(W)$. We set also $\pi^{(\sigma)}_a = 0$ on W together with $e^{(+1)}_a = e^{(-1)}_a = e_a$ and $\omega^{(+1)}_{ab} = \omega^{(-1)}_{ab} = \omega_{ab}$. Therefore, on the boundary, $\mathcal{F}^{(\sigma)}_{12} = \sigma\tau$ and $d\omega^{(\sigma)} = \tau$.

We then obtain the following total action

$$
\begin{aligned}
I_{tot}|_{b.c.} = I_6|^{(-1)}_{b.c.} + I_6|^{(+1)}_{b.c.} &= -\sum_{\sigma=\pm 1} c \left[\sigma \int_W \epsilon^{abcd} R_{ab} \wedge R_{cd} \right. \\
&\left. -2\int_W \epsilon^{abcd} R_{ab} \wedge e_c \wedge e_d + \sigma \int_W \epsilon^{abcd} e_a \wedge e_b \wedge e_c \wedge e_d \right] \\
&= 4c \int_W \epsilon^{abcd} R_{ab} \wedge e_c \wedge e_d
\end{aligned}
$$

which is the H-E action without any cosmological term [8].

The result of our construction is that, being able to map gravity in four dimension to an equivalent six dimensional topological theory, we find a specific $\mathbf{Z}_2$ symmetry which differs between the H-E action and the cosmological term and once it is implemented cancels the latter.

[8] In Section 2 we left aside the possibility that the six dimensional bundles are non trivial. In such a case we can not use Stoke's theorem directly on the action. Nonetheless, under infinitesimal variations of the connection, $\int_{Y_\varepsilon} \mathcal{F}^3$ reacts only to the change of the connection at the boundary. In fact $\delta \int_{Y_\varepsilon} \mathcal{F}^3 = 3 \int_{\partial Y_\varepsilon} \epsilon^{ABCDEF} \delta \mathcal{A}_{AB} \wedge \mathcal{F}_{CD} \wedge \mathcal{F}_{EF}$, where we used the Bianchi identity and Stoke's theorem. Under our boundary conditions, which then fix partly the freedom in varying the connection on the boundary, and summing over the two S-conjugate copies of the connection one finally gets that $\delta I_{tot}|_{b.c.} = \delta \left\{ 4c \int_W \epsilon^{abcd} R_{ab} \wedge e_c \wedge e_d \right\}$. From this we conclude that, since their equations of motion coincide, also in the case of non trivial bundles our topological theory is equivalent to gravity on W without the cosmological term. Let us stress that in this case the total action $I_{tot}|_{b.c.}$ possibly depends on global degrees of freedom in the bulk related to the structure of the two reduced $SO(3,3)$ bundles $\mathcal{X}^{(\sigma)}$.

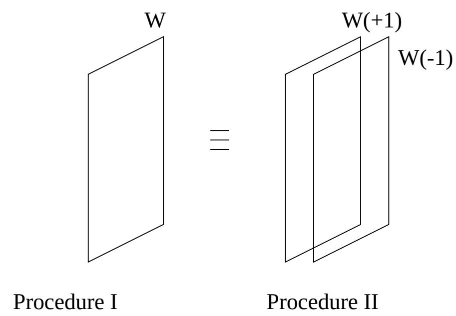

Fig. 1: *"Close enough" in a topological theory means homotopically equivalent. The two equivalent procedures.*

3.1 Inclusion of the matter sector

Now we want to speculate about a possible way to extend our construction to a slightly more realistic situation in which also quantum matter degrees of freedom are present on the four dimensional defect.

In the previous section we described a topological gauge theory in six dimensions which is equivalent to four dimensional gravity. In such a framework the cosmological term was canceled by the signature changing symmetry of the theory.

To implement our symmetry, we need a little further step. We want here to notice a trivial identification of two procedures in the above construction. The first procedure that we consider is the construction that we did in the previous section: we have chosen one four dimensional defect coupled to both the connections in a symmetric way via specific boundary conditions. Let us call this procedure I. Another possibility to obtain the same result is to consider two four dimensional defects W_σ, with $\sigma \pm 1$ each of them being coupled only to a single connection $\mathcal{A}^{(\sigma)}$ as in Section 2. We then consider them to be "close enough" in such a way that we are able to identify all degrees of freedom on them. But our six dimensional theory is topological and all non-zero bulk distances are physically equivalent from its point of view. Therefore the only possibility is that the two defects actually coincide as $W_\sigma = W$. This second construction leads exactly to the same result as before and we call it procedure II. The two equivalent procedures are described in *Fig.* 1. Let us now implement the equivalence of procedure I and II at a quantum level in an hamiltonian formulation. From the point of view of procedure II our Hilbert space can be considered to be of the form

$$\mathcal{H}_{phys} = \mathcal{H} \otimes \mathcal{H}/\!\sim ,$$

where $\sim$ is the following equivalence. Let any excitation on one of the two factor be created by its relative creation operator $a^\dagger$ and destroyed by a. Then, given any state $|\psi> \otimes |\eta>$, we implement the equivalence with procedure I, by representing the excitation in two conjugate equivalent ways as

$$|\psi> \otimes \left(a^\dagger|\eta>\right) \sim (a|\psi>) \otimes |\eta> \ .$$

Therefore we can choose in principle a picture to represent our physical Hilbert space by fixing a given state in, let's say, the second factor as $\mathcal{H}_{\bar{\eta}} = \mathcal{H} \otimes |\bar{\eta}>$. Since we are representing as equivalent particles in the first factor and anti-particles in the second one, the natural evolution operator is in the form $U = u \otimes u^\dagger$, where $u = \mathrm{e}^{ith}$ is the evolution operator on a single copy of our system. As a consequence, we find that total hamiltonian $H = h \otimes 1 - 1 \otimes h$ is odd under the $\mathbf{Z}_2$ permutation of the two factors of the Hilbert space.

Let us now note that the only representations which are stable under the time evolution are the ones for which $|\bar{\eta}>$ is an eigenvector for the hamiltonian h, i.e. $h|\bar{\eta}>= e_{\bar{\eta}}|\bar{\eta}>$. The vacuum state in a generic stable representation is represented by $|0> \otimes |\bar{\eta}>$, where $|0>$ is the vacuum state for h. Let us now ask for a $\mathbf{Z}_2$ symmetric vacuum. This condition reads as $|0> \otimes |\bar{\eta}> \sim \ |\bar{\eta}> \otimes |0>$ and is solved only by $|\bar{\eta}>= |0>$. We can now write down our total hamiltonian in the only symmetric representation $\mathcal{H}_s = \mathcal{H} \otimes |0>$ of $\mathcal{H}_{phys}$ as follows

$$H = h \otimes 1 - 1 \otimes h = (h - e_0) \otimes 1 \,,$$

where e_0 is the lowest eigenvalue of h and, by definition, $h|0>= e_0|0>$.

We can now calculate the vacuum energy of our system, which is its total contribution to the cosmological constant, to be zero as

$$E_{vac} = (<0| \otimes <0|) H (|0> \otimes |0>) = e_0 - e_0 = 0 \,.$$

4 Conclusions

In this letter we addressed the formulation of a new parity symmetry of the vacuum which, once it is implemented also at a quantum level, suggests a natural mechanism to cancel the cosmological constant. This is done by redrawing our world as a four dimensional defect in six dimensions and its gravitational degrees of freedom as governed by a topological theory in the bulk ambient space.

Let us speculate on the physical relevance, if any, of the six dimensional picture.

The obvious disadvantage of our model is that it may seem to be too artificial now. Indeed, one can say that, from four dimensional point of view, we are just finding a phenomenologically motivated possibility to prohibit the cosmological constant which is based on the observation that the latter has wrong transformation property under signature changing. The first goal of this paper was indeed to show a new mechanism for a simple symmetry to single out a vacuum whose energy is naturally zero. From this point of view, we can think that the six dimensional construction is just a mathematical device to render material our picture and that the two extra dimensions are metaphysical.

On the other hand, this symmetry of the vacuum turns out to be invisible to our world (i.e. we did not assumed any particular structure for the hamiltonian h in the previous section) and therefore, if these two extra – dimensions are metaphysiscal we do not have any a priori physical reason to insist on the reflection symmetry. Notice however that, if present, this symmetry turns out to be naturally stable under possible quantum corrections due to the fact that the two mirror sectors do not interact [9].

Another possibility is that our picture should be considered as a small building block which could be included in a deeper ultimate theory. A minimal way in this direction, from the point of view of the present picture, would be to give a six dimensional origin also to matter fields. This is beyond the goal of this letter, but we want to make one more final observation. A possible attitude could be to try to take more seriously the pure topological theory in the bulk and consider the degrees of freedom that we already have in the game. These are four dimensional defects coupled in the way that we described before. Suppose one lives on one of them W and feels the full six dimensional theory from there. If there are other dynamical defects W', he will have experience of them only trough their intersection with its own world $W \cap W'$. These intersections are two dimensional and might appear, in his given four dimensional world, as strings. This is due to the following decomposition $T(W)|_{W\cap W'} = T(W \cap W') \oplus \mathcal{N}(W')$ and to the facts that $T(W)$ is a $SO(1,3)$ bundle and $\mathcal{N}(W')$ a $SO(2)$ one. Therefore the only possible decomposition pattern for $T(W)$ is with $T(W \cap W')$ being a $SO(1,1)$ bundle. We see then that, from this point of view, it would seem possible to understand the six dimensional topological theory in which also the four dimensional defects became dynamical as a string theory when experienced from a single given defect.

[9] The four dimensional picture appearing at this point resambles at some extent the one discussed in [8]. We would like to thank E. Kiritsis and V. Mukhanov for bringing this paper to our attention after this paper was completed.

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Appendix: Some useful facts about $SO(3,3)$ and $so(3,3)$

$SO(3,3)$ is the group of rotations preserving the form $\eta = \mathrm{diag}(1,1,1,-1,-1,-1)$ as $R^t \eta R = \eta$ and with unit determinant. It admits an external automorphism given by the signature changing operator S which we identify with the adjoint action of the matrix element $S = \begin{pmatrix} 0 & \mathbf{1}_3 \\ \mathbf{1}_3 & 0 \end{pmatrix}$ as $\mathrm{S}R = SRS$. In fact, multiplying on the left and on the right side $R^t\eta R = \eta$ by S and using $S^2 = \mathbf{1}_6$ and $S^t = S$, we get $(\mathrm{S}R)^t(\mathrm{S}\eta)(\mathrm{S}R) = \mathrm{S}\eta$, where $\mathrm{S}\eta = S\eta S = -\eta$. This proves that S is a group automorphism for $SO(3,3)$. Notice that $\mathrm{S}^2 = \mathrm{id}$. Since S is not an element of $SO(3,3)$, this automorphism is called external. The S automorphism has naturally a counter part which acts on the $so(3,3)$ algebra. The S-invariant sub-algebra can be proved to be isomorphic to $gl(3)$. Since $gl(3)$ does not contain any $so(1,3)$ subalgebra, there is no S-invariant $so(1,3)$ subalgebra in $so(3,3)$ and, as groups are concerned, there is neither any S-invariant embedding of $SO(1,3)$ in $SO(3,3)$.

$so(1,3)$ subalgebras of $so(3,3)$ are specified as commutants with non invariant $so(2)$ subalgebras. There exists two inequivalent classes of them up to group conjugation. The two relative Cartan generators get in fact exchanged by S as $\mathrm{S}t_\pm = t_\mp$ (up to group conjugation). Notice that, given one of such $so(1,3)$ subalgebras one can slightly extend it to a one parameter family of $iso(1,3)$ algebras, parametrized by a $SO(2)$ angle, relative to the possible projection choices of half of the off block diagonal elements with respect to the $SO(2) \times SO(1,3)$ index decompositions.

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